

## LEARNING RATES AND MATHEMATICS ACHIEVEMENT IN CHILE

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In Chile, the poor quality of mathematics education can be traced in the low achievement levels reached by the students in standardized tests. The Chilean national assessment system, SIMCE (*Sistema de Medición de la Calidad de la Educación*), has consistently shown that students are able to do a lot less than the curricular expectations for their grade. For example, in the 8<sup>th</sup> grade, the mathematics results of SIMCE 2000 showed that only 45% of the students could correctly answer a one-step, straightforward problem that involved the use of fractions and operations (Ministerio de Educación [MINEDUC], 2001, p. 26). Considering that, according to the national curriculum, fractions should have been introduced in the 3<sup>rd</sup> grade (MINEDUC, 1980, 1999), it is a major concern that five years later more than half of the students are still unable to use them in the simplest applied contexts.

The poor performance level of the Chilean students has also been confirmed in the international arena. In the 1999 Trends in International Mathematics and Science Study (TIMSS), Chile ranked 35<sup>th</sup> out of the 38 countries participating in this study. The Chilean performance was significantly lower than Korea, Malaysia, and the Slovak Republic. Only South Africa, the Philippines, and Morocco had an average mathematics performance lower than Chile (Mullis et al., 2000, chap. 1).

Beyond the attained performance level, another important piece of information in assessing the quality of mathematics education is how much the students learn from one year to the other. The learning rate can be estimated following a cohort of students that is measured at the end of two consecutive years of instruction. An alternative to this longitudinal approach is to use data from cross-sectional studies that compare the performance of two groups of students at different points of the school cycle. For example, it is possible to compare the achievement levels of a current 7<sup>th</sup> and 8<sup>th</sup> grade classes and to use their differences in achievement as an estimation of learning rates.

TIMSS has provided evidence of learning rates in different countries. In the 1995 version of this study, nationally representative samples of 7<sup>th</sup> and 8<sup>th</sup> grade students were used to estimate learning rates in mathematics. Most of the countries had significant increases in their average mathematics performance between these two adjacent grades. For instance, the average score difference between the two grades was of 39 points in Spain, 31 points in Portugal, 30 points in Korea, and 16 points in Colombia (Beaton et al., 1996, p. 29).<sup>1</sup>

In Chile, little is known about the students' learning rates in mathematics. Chile did not participate in TIMSS 1995, and there are no studies that address learning rates using nationally

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<sup>1</sup> The TIMSS mathematics test had a standard deviation of 100 in the international achievement distribution.

representative samples of students. Moreover, the national assessment system does not follow cohorts of students and its design does not allow for comparing the performance of students in different grades.

Beyond the quantification of student gains, it is also of interest to learn about the type of knowledge and skills the students acquire in one year of instruction. From a policy standpoint, it is important to learn what factors are related to students' mathematics performance and learning gains. Recognizing that the schools (and the education systems) cannot be held accountable of all the factors that affect mathematics performance, attempts to assess school effectiveness usually take into account variables external to the school (e.g., parents' education, poverty levels) that have a known and strong effect on achievement. In Chile, several studies have shown that urban students with more educated parents and higher family income attain substantially higher mathematics scores than students from rural areas, with less educated parents and lower family income (Mizala and Romaguera, 2000; Ramírez, 2003).

Among the educational factors that can be affected through policy, the curriculum is a critical one. The curriculum affects performance by conditioning students' opportunities to learn mathematics. The Chilean mathematics curriculum, *Planes y Programas de la Educación General Básica*, was introduced in 1980, and was replaced by a new "reformed" framework and program of study in 2002.<sup>2</sup> The pre-reform curriculum listed the behavioral objectives to be reached by the students from grades 1-8. The schools were expected to "fill" the curricular framework with their own knowledge, and adapt it according to the needs of the community they served.

There is evidence of serious flaws that affected the pre-reform curriculum in Chile. Cox (1999) claimed that the provision of a general framework led to the impoverishment of the curriculum as it was implemented in the poorer schools. In fact, it seems that these schools never had the necessary resources to develop their own programs of study. Valverde (2004) compared the Chilean mathematics curriculum with the curriculum of other countries participating in TIMSS 1995. He noticed that the number of mathematics topics intended for instruction in the pre-reform curriculum were significantly fewer than the topics intended in the curriculum of the other participating countries.

TIMSS 1999 also provided evidence of a shallow mathematics curriculum. The mathematics contents and skills tested in TIMSS represent what the participating countries considered important to be learned at school up to the 8<sup>th</sup> grade. According to the test-curriculum matching analysis performed in this study, the Chilean mathematics curriculum had the lowest overlap with the test: only 58% of the problems in the test were considered appropriate for the 8<sup>th</sup> grade students. In contrast, 34 out of 38 participating countries indicated that three-quarters or more of the problems were appropriate according to their national curriculum (Mullis et al., 2000, appendix C).

The official curriculum is a guide for instruction, and it may differ in significant ways from the curriculum implemented in the mathematics classes. The discrepancies between the official and the implemented curriculum may arise from the necessary interpretation teachers do when reading the

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<sup>2</sup> *Planes y Programas de la Educación General Básica* was the curriculum in use when data was collected for this study.

former. Teachers have to decide the breadth and depth of the contents to be covered, the emphases on cognitive skills, and the difficulty level of the mathematics problems, among others.

The way the curriculum is implemented in classes determines the opportunities the students have to learn mathematics. In Chile, 72% of the 8<sup>th</sup> graders were taught by teachers that emphasized teaching numbers (i.e., whole numbers, fractions, decimals, percentages) (Mullis et al., 2000, p. 165). As noted in the Chilean report of TIMSS 1999, teaching toward the mastery of basic contents and skills may be jeopardizing the opportunities the students have to learn more advanced mathematics topics like algebra or geometry (MINEDUC, 2003).

Another aspect of the curriculum refers to the provision of differentiated instruction to accommodate students with different abilities. Supporters of differentiated instruction argue that it facilitates teaching, allows for more personalized instruction, and boosts achievement. Detractors of differentiated instruction claim that students' overall performance does not increase, that the achievement gap between high and low achievement students widens, that students are offered with unequal opportunities to learn, and that it produces increased social segregation in the schools (Davenport, 1993).

The evidence shows that the Chilean classes congregate students with a wide range of mathematics knowledge and skills (Ramírez, 2003). However, the Chilean curriculum did not address how to deal with students with different abilities (Mullis et al., 2000, p. 161). Accordingly, it was a school decision if the curriculum would be organized "the same for everybody" or some kind of differentiation would be introduced. To what extent were the Chilean schools providing differentiated instruction to the students? If so, was this practice related to increased mathematics performance?

This study used Chilean data from TIMSS 1999 to address some of the issues raised in this introduction. Four research questions guided the analyses:

1. How much mathematics were the students learning while progressing from the 7<sup>th</sup> to the 8<sup>th</sup> grade?
2. What mathematics content areas were emphasized in the 8<sup>th</sup> grade?
3. How did the schools organize the mathematics curriculum?
4. What curriculum variables were related to the variation in mathematics achievement among the 8<sup>th</sup> grade classes and schools?

## **Method**

### *Participants and Data Source*

In Chile, a nationally representative sample of 5,907 students in the 8<sup>th</sup> grade took the TIMSS test at the end of the school year of 1998 (November).<sup>3</sup> The students were nested in 185 schools, and

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<sup>3</sup> Chile and the other south-hemisphere countries collected their data at the end of the school year of 1998. The north-hemisphere countries did so at the end of their school year, around May 1999. In total, 38 countries participated in TIMSS 1999.

within each school one intact 8<sup>th</sup> grade class was sampled. The schools were sampled with probability proportional to size (PPS). Detailed information about the TIMSS sampling procedures can be found in Foy and Joncas (2000a, 2000b). As a national option, Chile also tested 6,063 students from the 7<sup>th</sup> grade classes of the same schools. Hence, two intact classes were sampled in each school.<sup>4</sup> The sampled classes were randomly selected among the classes available at each target grade.

All the students within the selected classes took the TIMSS test (mathematics and science) and answered the Students' Questionnaire (general version), providing background information about their parents' education and the resources they had at home. The teachers of the sampled students filled the Mathematics Teachers' Questionnaire, providing information about the topics covered in classes and emphases on different content areas. Finally, the principals from the sampled schools answered the School Questionnaire, which captured information related to how the mathematics curriculum was implemented in the schools. All this information is in the TIMSS 1999 International Database (2001) and in the databases of the Office of International Studies of the Chilean Ministry of Education. Detailed information about this and other technical aspects of TIMSS can be found in Martin, Gregory and Stemler (2000), and Gonzalez and Miles (2001).

### *Procedure*

The TIMSS questionnaires were scanned in search of questions related to the mathematics curriculum as it is implemented in the schools and classes, and related to students' family background. In some cases, the answers to these questions were combined into indices to create more valid and reliable measures of the constructs of interest. This section describes the variables and statistical procedures used to answer the research questions of this study. Table 1 provides further information about the variables used in this study.

### *Variables*

*Opportunities to learn index (OTL)*. This index was computed with the aim to know what mathematics topics were covered in the 8<sup>th</sup> grade classes. This index measured the number of topics the 8<sup>th</sup> grade teachers covered during the school year, as well as the depth (measured in teaching periods) of that coverage. The Teacher Questionnaire included a list of 34 mathematics topics that referred to the content areas covered in the mathematics test: fractions and number sense; measurement; geometry; algebra; proportionality; and data representation, analysis and probability. The mathematics teachers had to report if the topics were taught this year 1-5 periods (1 point), taught this year more than 5 periods (2 points), or were not yet taught (0 points).

For each teacher, a general opportunities to learn index (OTLg) was computed averaging the points across all 34 topics. The index was also computed separately for each content area. An index of opportunities to learn advanced mathematics topics (OTLa) was computed averaging all the content areas but excluding fractions and number sense topics. Paired samples t-test was used to compare the index of opportunities to learn fractions and number sense against the indices of opportunities to learn the other content areas.

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<sup>4</sup> The TIMSS 1999 sampling design targeted one grade only (grade 8 in most of the countries).

The mathematics teachers were also asked “what mathematics topics do you emphasize the most in your mathematics classes?” Their responses were recoded into a dummy variable where 0 = mainly numbers, and 1 = others (e.g., geometry, algebra). This variable was also used as an indicator of the mathematics curriculum implemented in the 8<sup>th</sup> grade classes.

TABLE 1: VARIABLES USED IN THE STUDY

Variable label (name)	Variable Description
Outcome	
Mean mathematics 8 (Math8)	Mean mathematics scores at the 8 <sup>th</sup> grade
Covariates	
Mean mathematics 7 (Math7)	Mean mathematics scores at the 7 <sup>th</sup> grade
Socio-economic index (SEI)	Aggregated variable derived from linear combination of: father and mother education, number of books in the home, and possessions at home
Curriculum Related Variables	
Opportunities to learn general (OTLg)	Opportunities to learn general index: Based on a list of 34 topics included in the mathematics test. The topics represented 6 content areas: fractions and number sense; measurement; geometry; proportionality; algebra; and data representation, analysis and probability
Opportunities to learn advanced (OTLa)	Opportunities to learn advanced mathematics index: Based on a list of 22 topics included in the mathematics test. The topics represented 5 content areas: measurement; geometry; proportionality; algebra; and data representation, analysis and probability
Subject matter emphasis (SME)	Dummy variable for subject matter emphasis: 0=mainly numbers; 1=others (i.e., geometry; algebra; combined geometry and algebra; combined geometry, algebra and numbers; or others)
School curriculum (SCHCURR)	Dummy variable for written statement of curriculum: 0=no; 1=yes
Same content at different levels (CURR1)	Dummy variable for teaching same content at different difficulty levels: 0=no; 1=yes
Ability grouping (CURR2)	Dummy variable for students grouped by ability within mathematics classes: 0=no; 1=yes
Enrichment math (CURR3)	Dummy variable for offering of enrichment mathematics: 0=no; 1=yes
Remedial math (CURR4)	Dummy variable for offering remedial mathematics: 0=no; 1=yes

*The school mathematics curriculum.* Five questions from the School Questionnaire were used to characterize the curriculum as it is organized in the schools:

1. Does your school have its own written statement of the curriculum content to be taught (i.e., other than the national curriculum guides)?
2. All classes study similar content, but at different levels of difficulty (e.g., setting or streaming)
3. Students are grouped by ability within their mathematics classes
4. Enrichment mathematics is offered
5. Remedial mathematics is offered

*Previous achievement level.* This study paired the mean mathematics scores of a 7<sup>th</sup> and 8<sup>th</sup> grade classes within the same schools. It was assumed that the 7<sup>th</sup> grade scores were equivalent to the scores that a current 8<sup>th</sup> grade class would have obtained if tested a year early. Evidence in support of this assumption is that the Chilean schools do not use to track 8<sup>th</sup> grade students into different type of classes. The main weakness of this approach, though, is its failure to recognize “good” and “bad” years of schools due to differences in cohorts of students. In the predictive models (see next section), the 8<sup>th</sup> grade mean scores for the classes and schools were used as the outcome and the 7<sup>th</sup> grade mean scores as a covariate.

*Socio-economic index (SEI).* The socio-economic index consisted of a linear combination and further aggregation of the students’ responses to three questions regarding their home background characteristics: father and mother education, number of books in the home, and possessions at home (e.g., computer, refrigerator). The mean socio-economic index for the class was used as an indicator of the average socio-economic background of the community served by the schools. This index was also used as a covariate in the predictive models.

### *Modeling Mathematics Achievement*

TIMSS 1999 used a clustered sampling design where students were nested in classes and classes were nested in schools. Since only one class per grade was tested at the 8<sup>th</sup> grade, the differences among the schools cannot be disentangled from the differences among the 8<sup>th</sup> grade classes from the same schools. As a consequence, the schools and the classes are confounded in one unit of analysis that will be referred as the school/class unit or level.

Considering the clustered nature of the data, a two-level hierarchical linear model was used to model mathematics achievement. At the students-within-class level, the students’ mathematics scores at the 8<sup>th</sup> grade were used as the outcome variable (five plausible values). At the school/class level, the mean mathematics scores of the 8<sup>th</sup> grade classes served as the outcome. To estimate the proportion of achievement variance that was between the schools/classes, an unconditional model was created (Equations 1 and 2). Since this study aimed to predict achievement differences among the schools/classes, Equation 1 was kept the same across the different models tested at the school/class level.

Six predictive models were tested at the school/class level. In Model 1 only the 7<sup>th</sup> grade mean scores were entered in the equation (Equation 3). Model 2 encompassed Model 1 plus a block of variables related to the opportunities the students had to learn the topics evaluated in the mathematics test. The predictors were opportunities to learn (general), opportunities to learn (advanced), and subject matter emphasis (Equation 4). Model 3 encompassed Model 2 plus a block of variables related to the provision of differentiated instruction: same content delivered at different difficulty levels, ability grouping, enrichment, and remedial instruction (Equation 5). Model 4 encompassed Model 3 plus a variable measuring if the schools had their own written statement of the mathematics curriculum (Equation 6). Model 5 encompassed Model 4 plus the aggregated socio-economic index (Equation 7). Model 6 was restricted to the variables that made a significant partial contribution to the outcome variance (Equation 8).

Unconditional model:

$$y_{ij} = \beta_{0j} + r_{ij} \quad (1)$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \quad (2)$$

Predictive models:

$$y_{ij} = \beta_{0j} + r_{ij} \quad (1)$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{math7}_j) + u_{0j} \quad (3)$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{math7}_j) + \gamma_{02}(\text{OTLg}_j) + \gamma_{03}(\text{OTLa}_j) + \gamma_{04}(\text{SME}_j) + u_{0j} \quad (4)$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{math7}_j) + \gamma_{02}(\text{OTLg}_j) + \gamma_{03}(\text{OTLa}_j) + \gamma_{04}(\text{SME}_j) + \gamma_{05}(\text{curr1}_j) + \gamma_{06}(\text{curr2}_j) + \gamma_{07}(\text{curr3}_j) + \gamma_{08}(\text{curr4}_j) + u_{0j} \quad (5)$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{math7}_j) + \gamma_{02}(\text{OTLg}_j) + \gamma_{03}(\text{OTLa}_j) + \gamma_{04}(\text{SME}_j) + \gamma_{05}(\text{curr1}_j) + \gamma_{06}(\text{curr2}_j) + \gamma_{07}(\text{curr3}_j) + \gamma_{08}(\text{curr4}_j) + \gamma_{09}(\text{schcurr}_j) + u_{0j} \quad (6)$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{math7}_j) + \gamma_{02}(\text{OTLg}_j) + \gamma_{03}(\text{OTLa}_j) + \gamma_{04}(\text{SME}_j) + \gamma_{05}(\text{curr1}_j) + \gamma_{06}(\text{curr2}_j) + \gamma_{07}(\text{curr3}_j) + \gamma_{08}(\text{curr4}_j) + \gamma_{09}(\text{schcurr}_j) + \gamma_{010}(\text{SCI}_j) + u_{0j} \quad (7)$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{math7}_j) + \gamma_{03}(\text{OTLa}_j) + \gamma_{09}(\text{schcurr}_j) + \gamma_{010}(\text{SCI}_j) + u_{0j} \quad (8)$$

Where:

$y_{ij}$  = Mathematics score of student  $i$  in school  $j$

$\beta_{0j}$  = Mean mathematics score for school/class  $j$

$\gamma_{00}$  = Intercept (grand mean mathematics score)

$\gamma_{01} \dots \gamma_{010}$  = Partial regression coefficients for the predictors and covariates

$(\text{math7}_j)$ ,  $(\text{OTLg}_j)$ , ...,  $(\text{SME}_j)$ ,  $(\text{curr1}_j)$ , ...  $(\text{schcurr}_j)$ ,  $(\text{SCI}_j)$  = Value of the predictors and covariates for school/class  $j$

$r_{ij}$  = Random error term for student  $i$  in school/class  $j$

$u_{0j}$  = Random error term for school/class  $j$

All the variables were standardized ( $M = 0$  and  $SD = 1$ ) so that direct comparisons can be made among their partial regression coefficients. All the results were weighted to account for the differential probability of the students and schools of being selected in the TIMSS sample.

In reading the results attention to two caveats is recommended. First, the sample selected for TIMSS is a good sample of students, but not necessarily an optimal sample of schools. Schools were selected with probability proportional to the number of 8<sup>th</sup> graders in the schools. Hence, it is expected that there is a greater number of large schools in the sample. Second, the predictive models were run using a restricted sample as a consequence of missing cases. From the original 185 schools, only 119 (64%) met the criteria of no missing data at the school/class level. As a consequence of losing schools, the number of students included in the analyses was halved. Accordingly, caution is recommended in generalizing the results at the country level.

## Results

In this section, the results are presented following the order of the four research questions that guided the analyses. First, students' mathematics learning rates while progressing from the 7<sup>th</sup> to the 8<sup>th</sup> grades are presented. Second, the focus turns to the coverage received by the mathematics topics in the 8<sup>th</sup> grade. Third, the Chilean schools are characterized by how they organize the mathematics curriculum to address the needs of students with different abilities. The fourth and last section presents several predictive models of mathematics achievement.

### *How Much Mathematics Were the Students Learning While Progressing from the 7th to the 8th Grade?*

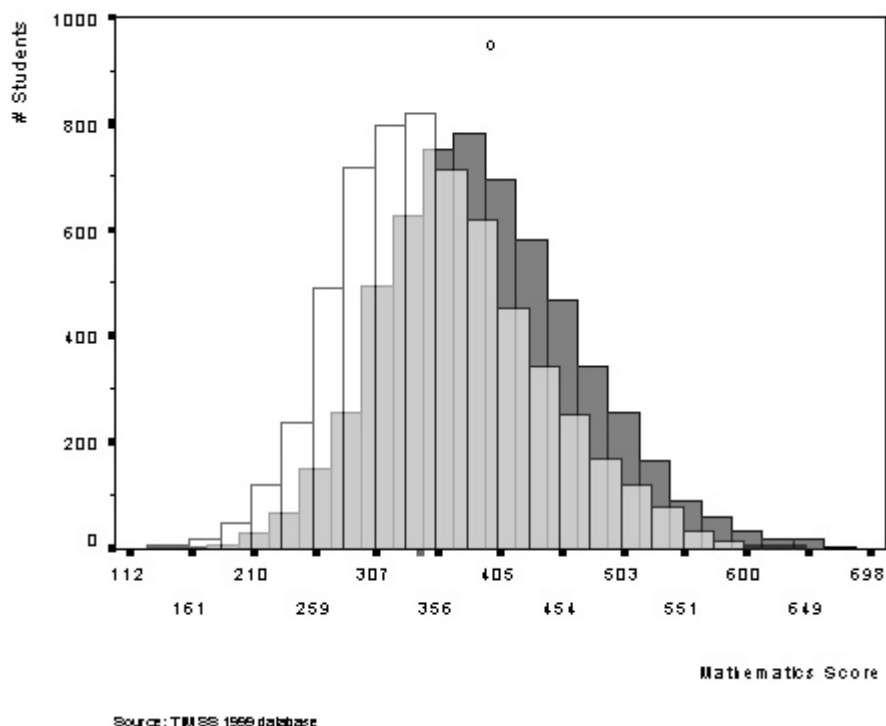
Figure 1 compares the distribution of mathematics scores for the 7<sup>th</sup> and 8<sup>th</sup> grade students. At the upper grade, students averaged 392 points, whereas those in the lower grade 359. The 33 points of difference between the grades was equivalent to 0.4 standard deviations in the Chilean achievement distribution. A one-way analysis of variance (ANOVA) showed that this was a significant increase in mathematics achievement,  $F(1, 11967) = 569, p < .0005$ .

On average, students in the 8<sup>th</sup> grade answered correctly six more mathematics problems (out of a pool of 162 problems in the test), compared to the 7<sup>th</sup> graders. Nearly one-third of the 8<sup>th</sup> graders (35%) obtained scores below the 7<sup>th</sup> grade mean. A similar proportion of 7<sup>th</sup> graders (30%) scored at or above the grade 8<sup>th</sup> mean. The ratio of achievement variance that was between the grades to the total achievement variance was equal to 5%. The remaining 95% corresponded to differences in achievement within the grades.

TIMSS 1999 identified four cut-points (benchmarks) in the international mathematics scale: percentiles 25, 50, 75, and 90. The students' performance at these benchmarks was described. Since there were four cut-points, five achievement groups could be identified: below percentile 25 (Group 1), between percentiles 25-50 (Group 2), between percentiles 50-75 (Group 3), between percentiles 75-90 (Group 4), and above percentile 90 (Group 5). These groups were used to classify the Chilean 7<sup>th</sup> and 8<sup>th</sup> graders according to their performance on the mathematics test (Figure 2). Most of the students fell in Group 1 (71% of the 7<sup>th</sup> graders and 55% of the 8<sup>th</sup> graders). Students in Group 1 did *not* provide enough evidence of mastering the easiest problems on the test. Consequently, it was not possible to describe what they could actually *do*. From this default category, it can just be inferred that these students were *not* even able to do basic computations with whole numbers.



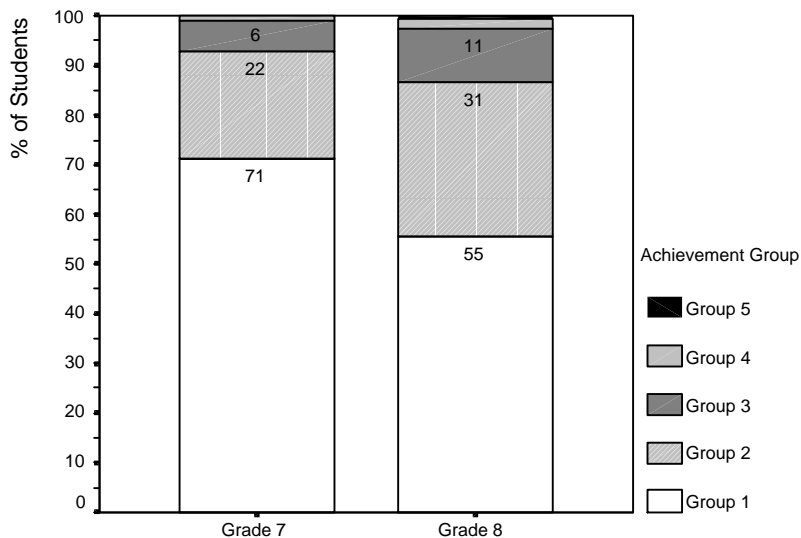
FIGURE 1. VARIATION OF MATHEMATICS ACHIEVEMENT, WITHIN AND AMONG THE 7TH AND 8TH GRADES. FIVE PERCENT OF THE VARIATION IN MATHEMATICS SCORES WAS BETWEEN GRADES; THE OTHER 95% CORRESPONDED TO THE VARIATION IN SCORES WITHIN THE GRADES. GRADE 7 DISTRIBUTION:  $M = 359$ ,  $SD = 75$ ,  $N = 6,063$  (FULL SAMPLE). GRADE 8 DISTRIBUTION:  $M = 392$ ,  $SD = 77$ ,  $N = 5,907$  (FULL SAMPLE).



At the 8<sup>th</sup> grade, 31% of the students were classified in Group 2. These students “...can add, subtract, and round with whole numbers. When there are the same number of decimal places, they can subtract with multiple regrouping. Students can round whole numbers to the nearest hundred. They recognize some basic notation and terminology” (Mullis et al., 2000, p. 42). Figure 3 shows an example of the type of problems students in Group 2 or higher were likely to answer correctly.

At grade 8, only 14% of the students fell in Group 3 or above; most of these students (11%) provided evidence of being able to “apply basic mathematics knowledge in straightforward situations. They can add or subtract to solve one-step word problems involving whole numbers and decimals; identify representations of common fractions and relative sizes of fractions; solve for missing terms in proportions, recognize basic notions of percents and probability; use basic properties of geometric figures; read and interpret graphs, tables, and scales; and understand simple algebraic relationships” (Mullis et al., 2000, p. 43). Figure 4 provides an example of the type of problems students in achievement Group 3 or higher were likely to answer correctly. The comparison of Figures 3 and 4 is helpful to see the kind of progress the student made while moving from achievement Group 2 to 3.

FIGURE 2. DISTRIBUTION OF MATHEMATICS SKILLS AT THE 7TH AND 8TH GRADES. GROUP 1 REPRESENTS THE STUDENTS WHO SCORED BELOW PERCENTILE 25 IN THE INTERNATIONAL ACHIEVEMENT DISTRIBUTION, GROUP 2 THOSE WHO SCORED BETWEEN PERCENTILES 25-50, GROUP 3 BETWEEN PERCENTILES 50-75, GROUP 4 BETWEEN PERCENTILES 75-90, AND GROUP 5 ABOVE PERCENTILE 90.



Source: TIMSS 1999 database

FIGURE 3. EXAMPLE OF A PROBLEM THAT STUDENTS IN ACHIEVEMENT GROUP 2 OR HIGHER WERE LIKELY TO ANSWER CORRECTLY. IN CHILE, THIS PROBLEM WAS CORRECTLY ANSWERED BY 54% OF THE 7TH GRADERS AND BY 65% OF THE 8TH GRADERS. AT THE 8TH GRADE, THIS PROBLEM WAS CORRECTLY ANSWERED BY 93% OF THE STUDENTS IN HUNGARY AND BY 88% OF THE STUDENTS IN MALAYSIA. THIS PROBLEM MEASURED FRACTIONS AND NUMBER SENSE, AND REQUIRED THE STUDENTS TO ROUND TWO THREE-DIGIT NUMBERS TO ESTIMATE THE FINAL RESULT. WHILE ROUNDING AND ESTIMATING WERE NOT OBJECTIVES IN THE CHILEAN CURRICULUM, OPERATING WITH NATURAL NUMBERS WAS AN EXPLICIT OBJECTIVE FOR ALL GRADES IN PRIMARY SCHOOL (MINEDUC, 1980, 1999).

The sum  $691 + 208$  is closest to the sum:

- A.  $600 + 200$
- B.  $700 + 200$  \*
- C.  $700 + 300$
- D.  $900 + 200$

FIGURE 4. EXAMPLE OF A PROBLEM THAT STUDENTS IN ACHIEVEMENT GROUP 3 OR HIGHER WERE LIKELY TO ANSWER CORRECTLY. IN CHILE, THIS PROBLEM WAS CORRECTLY ANSWERED BY 38% OF THE 7TH GRADERS AND BY 48% OF THE 8TH GRADERS. IN KOREA, 84% OF THE 8TH GRADERS GOT IT CORRECT, AS WELL AS 78% OF THE STUDENTS IN HUNGARY, THE SLOVAK REPUBLIC AND MALAYSIA. WHILE ROUNDING NUMBERS AND ESTIMATING WERE NOT OBJECTIVES OF THE CHILEAN MATHEMATICS CURRICULUM, THE USE OF THE FOUR BASIC OPERATIONS TO WORK WITH NATURAL NUMBERS WAS AN OBJECTIVE FOR ALL GRADES IN THE PRIMARY SCHOOL (MINEDUC, 1980, 1999)

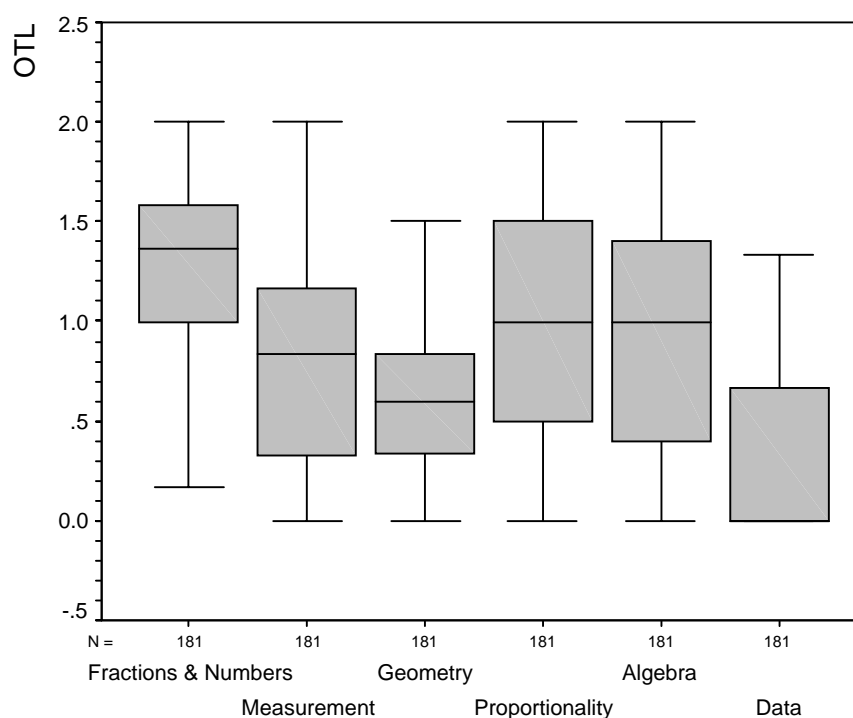
There are 68 rows of cars in a parking lot. Each row has 92 cars. Which of these would give the closest estimate of the total number of cars in the parking lot?

- A.  $60 \times 90 = 5400$
- B.  $60 \times 100 = 6000$
- C.  $70 \times 90 = 6300$  \*
- D.  $70 \times 100 = 7000$

### *What Mathematics Content Areas Were Emphasized in the 8th Grade?*

The opportunities to learn index (OTL) for the five content areas covered in the TIMSS mathematics test is depicted in Figure 5. As shown by the boxplots, the index value changed drastically depending upon the content area under analysis. Fractions and number sense was the most widely covered area ( $M_{OTL_n} = 1.30$ ), while data representation was the least covered one ( $M_{OTL_d} = 0.46$ ). The index value for fractions and number sense was significantly higher than the index value for the other content areas ( $p < .0005$ ).

FIGURE 5. OPPORTUNITIES TO LEARN INDEX (OTL) BY MATHEMATICS CONTENT AREAS. OTL EQUALS ZERO MEANS THAT THE CONTENT AREA WAS NOT COVERED DURING THE 8<sup>TH</sup> GRADE SCHOOL YEAR. OTL EQUALS ONE MEANS THE CONTENT AREA WAS COVERED ONE TO FIVE PERIODS. OTL EQUALS 2 MEANS THAT THE CONTENT AREA WAS COVERED IN MORE THAN FIVE PERIODS. THE BOXPLOTS SHOW THE MEDIAN, INTERQUARTILE RANGE, AND THE 5<sup>TH</sup> AND 95<sup>TH</sup> PERCENTILES OF THE INDEX DISTRIBUTION. BASED ON  $N = 181$  MATHEMATICS TEACHERS THAT PROVIDED VALID DATA TO COMPUTE THE INDICES.



It is also notable how much the 8th grade teachers varied in their reports of curricular coverage within a content area. In three out of the five areas, the whole index range was reported: while some mathematics teachers did not cover at all measurement, proportionality, and algebra, others reported to have covered these areas during more than five instructional periods.

### *How Was the Mathematics Curriculum Organized in the Schools?*

Eighteen percent of the schools reported to have developed their own written statement of the mathematics curriculum; almost all these schools were private (elite-paid or private-subsidized). These schools attained mathematics scores substantially higher than the schools that did not have their own

curriculum (80 points more, roughly 1.6 standard deviations in the schools' mean achievement distribution).

Among the four instructional differentiation strategies, offering remedial mathematics was the most widely used by the schools (81%), followed by classes studying the same content but at different levels of difficulty (67%). Ability grouping and enrichment were offered by around one-fourth of the schools (27% and 24%, respectively). Only the schools that offered enrichment mathematics presented significantly higher scores than those that did not offer it ( $p < .05$ ). Schools with ability grouping had significantly lower mean scores than schools that did not use ability grouping. Schools offering the same content at different difficulty levels also had significantly lower mean mathematics scores than schools that did not offer it ( $p < .05$ ).

Because of the correlational nature of this study, these associations should not be taken as implying causality. Is it that schools with higher mean achievement prefer to offer enrichment mathematics, or is it that offering enrichment mathematics leads to increased achievement? The same kind of questions can be posed regarding the other instructional strategies. In the following section, the study aims to evaluate the effect of these instructional strategies on achievement.

#### *What Curriculum Variables Were Related to the Variation in Mathematics Achievement Among the 8th Grade Schools/Classes?*

The unconditional model showed that 36% of the variation in mathematics scores was among the schools/classes. The other 64% corresponded to achievement differences among classmates who attended the same classes and schools. Table 2 shows the results of six regression models run at the school/class level. Model 1 shows that 78% of the differences in achievement among the schools/classes can be accounted for by the mean mathematics scores in the grade 7 class ( $p < .0005$ ).

The next models attempted to explain the variance *not* accounted for by the 7<sup>th</sup> grade mean scores (residual variance). Model 2 tested the effect of three variables related to emphases on mathematics content areas: the general index of opportunities to learn (OTLg), the index of opportunities to learn advanced mathematics topics (OTLa), and a dummy variable for subject matter emphasis (mainly numbers versus others). This model accounted for 79% of the outcome variance, one percentage point more than the previous model. The index of opportunities to learn advanced mathematics topics (OTLa) had the strongest partial relationship with achievement ( $p < .05$ ).

Model 3 tested a new equation that included the variables in blocks 1 and 2, plus four variables measuring the delivery of differentiated mathematics instruction: offering the same content at different difficulty levels, ability grouping, enrichment mathematics, and remedial mathematics. None of the variables in block 3 accounted for additional outcome variance, after controlling statistically for the effect of the other variables in the model. No increase was detected in the percent of accounted variance (79%).

Model 4 considered the effect of the variables in blocks 1, 2, and 3, together with the schools "having their own written statement of the mathematics curriculum." This variable made a strong unique contribution to the outcome variance, after controlling statistically for the effect of the other variables in the model ( $p < .0005$ ). Having a written statement of the curriculum accounted for an additional 2% of the outcome variance.

TABLE 2: PREDICTORS OF SCHOOLS/CLASSES' MEAN MATHEMATICS SCORES IN THE 8<sup>TH</sup> GRADE

	Model 1 (Block 1)	Model 2 (Blocks 1-2)	Model 3 (Blocks 1-3)	Model 4 (Blocks 1-4)	Model 5 (Blocks 1-5)	Model 6 (Restricted)
Between schools/classes variance accounted for by model	78%	79%	79%	81%	83%	83%
	Betas	Betas	Betas	Betas	Betas	Betas
Mean mathematics scores at grade 8 (intercept)	391	391	391	391	391	391
BLOCK 1: PREVIOUS LEARNING						
Mean mathematics score at grade 7	44.24***	43.90***	42.83***	37.96***	27.79***	27.34***
BLOCK 2: OPPORTUNITIES TO LEARN						
Opportunities to learn general index (OTLg)		-12.36	-11.27	-11.86	-7.42	
Opportunities to learn advanced index (OTLa)		15.90**	14.33*	14.07*	10.65	4.05*
Subject matter emphasis (SME)		0.38	0.09	-0.28	-0.69	
BLOCK 3: DIFFERENTIATION OF CURRICULUM						
Same content at different difficulty levels (CURR1)			-1.89	-1.83	-1.83	
Students grouped by ability within mathematics classes (CURR2)			-1.87	-2.01	-1.81	
Enrichment mathematics is offered (CURR3)			3.33	3.29	2.91	
Remedial mathematics is offered (CURR4)			1.63	0.66	0.43	
BLOCK 4: SCHOOL CURRICULUM						
Written statement of curriculum (SCHCURR)				8.57***	7.71***	7.69***
BLOCK 5: SOCIAL CONTEXT						
Socio-economic index (SEI)					12.97***	14.62***

*Note.* Models weighted with HOUWGT variable. All predictors were standardized and grand-mean centered. Method: Restricted maximum likelihood with robust standard errors. Significant alpha levels: \*\*\* $p < .01$ ; \*\* $p < .05$ ; \* $p < .10$ .

Model 5 evaluated if the previous findings held true after controlling for the impact of the socio-economic index (SEI). This covariate made a strong unique contribution to the achievement variance ( $p < .0005$ ), after controlling statistically for the effect of the other variables in the model. This model increased the amount of variance explained from 81% to 83%.

Model 6 included only the variables most consistently associated with the achievement outcome: if the schools had their own written statement of the curriculum, the index of opportunities to learn advanced mathematics topics, the grade 7 mean scores and the socio-economic index. All these variables made a unique significant contribution to the outcome, and accounted for 83% of the achievement variance.

## Discussion

This study analyzes the mathematics knowledge and skills of a nationally representative sample of Chilean students in the 7<sup>th</sup> and 8<sup>th</sup> grades. Using data from the 1999 Trends in International Mathematics and Science Study (TIMSS 1999), this paper describes the relative emphases teachers put on different mathematics content areas, provides information on how the schools organize the mathematics curriculum, and on how these curricular strategies relate to achievement. In this section,

the main findings of the study are presented and discussed in the context of the relevant literature. Some policy implications are raised.

On average, the Chilean 8<sup>th</sup> graders scored 33 points higher than the 7<sup>th</sup> graders in the TIMSS 1999 mathematics test, which was a significant increase in mathematics performance (equivalent to 0.4 standard deviations in the achievement distribution of the Chilean students). This difference is similar to the one obtained by countries like Portugal, Canada and Korea in TIMSS 1995 (Beaton et al., 1996). These findings suggest that the Chilean students have a learning rate similar to the students from countries with a somewhat higher economic development.

Nevertheless, if the Chilean learning rate (i.e., the 33 points of difference between the two adjacent grades) were projected beyond the 8<sup>th</sup> grade, it is clear that the Chilean students are lagging three years behind compared to the students from countries performing near the international average in TIMSS 1999. For instance, while Chile averaged 392 points in the mathematics test, New Zealand got 491 points, and Italy 479.

Despite the significant improvements, the 7<sup>th</sup> and 8<sup>th</sup> graders share a lot in common in terms of their mathematics knowledge and skills. While 5% of the variation in mathematics achievement lies between the grades, 95% is within-grades variation. Nearly one-third of the 8<sup>th</sup> graders had mathematics scores below the 7<sup>th</sup> grade mean, and a similar proportion of 7<sup>th</sup> graders scored above the 8<sup>th</sup> grade mean.

The Chilean students are substantially delayed in the type of mathematics knowledge and skills they manage, both according to the international benchmarks set by TIMSS and, most importantly, according to the national objectives set by the Chilean curriculum. Only a marginal number of the 8<sup>th</sup> grade students (around 5%) perform according to the curricular expectations for their grade. These findings are consistent with the information provided by the national assessment system, SIMCE (MINEDUC, 2001).

In both grades, most of the students did *not* provide enough evidence of being able to do basic computations with whole numbers—adding, subtracting and rounding. This was the case for 71% of the 7<sup>th</sup> graders and 55% of the 8<sup>th</sup> graders. This is an unexpected finding since performing basic computations with whole numbers should be taught starting from grade 1 according to the Chilean curriculum (MINEDUC, 1980, 1999). Doing computations with whole numbers is also the skill that teachers emphasize the most in their mathematics classes. These findings suggest important inefficiencies in the teaching and learning processes.

What role is the mathematics curriculum playing in this situation? Research has shown that the curriculum as it is implemented in the classrooms may differ widely from the official curriculum. This is not surprising since teachers interpret and adapt the official curriculum according to their own pedagogical styles and beliefs, and also according to the characteristics of their students. When the Chilean teachers were asked about what mathematics areas they covered during the 8<sup>th</sup> grade school year, they reported that fractions and number sense was the most widely covered. Other content areas like measurement, geometry, proportionality, algebra and data received significantly less attention in classes.

According to the Chilean mathematics curriculum, fractions and number sense should have been emphasized during the first cycle of basic education (grades 1 to 4). By the time the students reach the 8<sup>th</sup> grade, the emphasis should have shifted toward most advanced content areas like geometry, algebra, and data representation (MINEDUC, 1980, 1999). The major emphasis that fractions and number sense continues to receive at the 8<sup>th</sup> grade suggests a substantial gap between the curricular intentions (as specified by the national curriculum) and the implemented curriculum (as reported by the teachers). This gap should be a major concern for the educational policy in Chile. Considering that a new 8<sup>th</sup> grade mathematics curriculum was introduced in 2002, strategies aimed to support and closely monitor its implementation are strongly recommended.

While the official curriculum put pressure on the teachers to cover more advanced mathematics topics, it seems that other factors have more weight when deciding what to teach. The lack of preparation to teach most advanced topics may be one of them. The Chilean teachers are among those who report the lowest levels of confidence in their preparation to teach mathematics, as compared to the teachers from other countries (Mullis et al., 2000, p. 192).

Another element that might be playing a role in what teachers teach is the previous mathematics level of their students. As shown in this study, there is an important gap between what the students can actually do and the curricular expectations for their grade. It is highly probable that this delay is pushing the teachers to keep reviewing basic topics like fractions and number sense. While this may be appropriate in some cases, it is necessary to highlight the trade-offs of this practice. The Chilean students are not being provided with enough opportunities to learn the more advanced mathematics topics stated in the national curriculum and that are regularly assessed in standardized achievement tests.

An index of opportunities to learn advanced mathematics topics was used to measure the time teachers spend teaching measurement, proportionality, geometry, algebra, data representation, analysis and probabilities. According to expectations, this index was significantly related to achievement, after controlling statistically for the effect of the socio-economic background of the students and for the previous achievement level of the classes. These findings suggest that schools working in similar social contexts and whose students possess a similar repertoire of mathematics knowledge and skills are likely to attain higher performance if the teachers cover more advanced mathematics topics.

The development of programs of study by the schools has been an explicit policy of the educational reform in Chile (MINEDUC, 1998, p. 51). However, only 18% of the schools reported having their own written statement of the curriculum and almost all of these schools were private (elite-paid and private-subsidized). The schools that have their own written statement of the curriculum that goes beyond the specifications of the national curriculum attained significantly higher mean mathematics performance than schools that do not have their own curricular statements, after controlling statistically for previous achievement and socio-economic level. Following Cox (1999), it is likely the lack of technical and material resources is precluding the public (and poorer) schools to develop their own curriculum and instructional strategies. Ensuring that the public schools have the necessary support to adapt the official curriculum to their own local contexts and needs seems to be a recommendable policy.

This study also tested the effect of four different ways to organize the curriculum within the schools: same content taught at different difficulty levels, students grouped by ability within the

mathematics classes, enrichment mathematics, and remedial mathematics. None of these strategies was associated with higher mean scores, after controlling statistically for previous achievement and socio-economic level. In other words, the type of instruction offered by the schools –unified versus differentiated– does not seem to explain the wide differences in average performance among similar schools.

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